ECEN 462 Exam #2

Oct. 21 – Oct. 28

Fall 2009

Prof. Stephen M. Schultz – 422-1693

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Instructions – Please Read

1. Closed book and closed notes
2. *All work should start from an equation in the appendix NOT start from an equation that you have memorized.*
3. **Graphing Calculator Allowed**
4. Ruler and compass
5. This exam consists of 7 problems + appendix. These are all work out problems so be sure to show your work.
6. The first 5 problems are slightly shorter and are worth 12 points each. The last 2 problems are worth 20 points each.
7. Please make any standard class assumptions *unless otherwise indicated*. Some standard assumptions are that if $\mu_r>1$ then $\varepsilon_r=1$ and vice versa, propagation is in the z-direction, mks units, perfect conductors for all metals, charge free, lossless materials, linear, time invariant, etc… Any other assumptions should be stated.

Appendix

$\vec{D} = \varepsilon_o \,\bar{\bar{\varepsilon}}\, \vec{E}$

Permittivity tensor for uniaxial material $\bar{\bar{\varepsilon}} = \varepsilon_o \begin{bmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_E^2 \end{bmatrix}$

Permittivity tensor for biaxial material $\bar{\bar{\varepsilon}} = \varepsilon_o \begin{bmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{bmatrix}$

Permittivity tensor for gyrotropic material $\bar{\bar{\varepsilon}} = \varepsilon_o \begin{bmatrix} \varepsilon_R & j\kappa & 0 \\ -j\kappa & \varepsilon_R & 0 \\ 0 & 0 & \varepsilon_R \end{bmatrix}$

Propagation constant for a right-handed circular polarization state traveling in the z-direction in a gyrotropic material $k_R = \dfrac{2\pi}{\lambda}\sqrt{\varepsilon_R - \kappa}$

Propagation constant for a left-handed circular polarization state traveling in the z-direction in a gyrotropic material $k_L = \dfrac{2\pi}{\lambda}\sqrt{\varepsilon_R + \kappa}$

Rotation angle for linear polarization traveling through a slab of gyrotropic material $\phi = -\dfrac{1}{2}(k_R - k_L)d$ or $\phi = V\,B\,d$

Permittivity tensor of several common materials:

Calcite: $\bar{\bar{\varepsilon}} = 8.854\times10^{-12} \begin{bmatrix} 1.66^2 & 0 & 0 \\ 0 & 1.66^2 & 0 \\ 0 & 0 & 1.49^2 \end{bmatrix}$

Crystalline Quartz: $\bar{\bar{\varepsilon}} = 8.854\times10^{-12} \begin{bmatrix} 1.54^2 & 0 & 0 \\ 0 & 1.54^2 & 0 \\ 0 & 0 & 1.55^2 \end{bmatrix}$

Potassium Titanyl Phoshate (KTP): $\bar{\bar{\varepsilon}} = 8.854\times10^{-12} \begin{bmatrix} 1.76^2 & 0 & 0 \\ 0 & 1.77^2 & 0 \\ 0 & 0 & 1.87^2 \end{bmatrix}$

Free-space Green's function: $g(r) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}$

Magnetic Vector potential: $\vec{A}(r) = \frac{\mu}{4\pi}\int_{v'} \vec{J}(r')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}dv'$

Far field magnetic vector potential: $\vec{A}_{ff}(r) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\int_{v'} \vec{J}(r')\, e^{-jk\hat{r}\cdot\vec{r}'}dv'$

Far field electric field: $\vec{E}_{ff} = j\omega\,\hat{r}\times\hat{r}\times\vec{A}_{ff}$

Far field magnetic field: $\vec{H}_{ff} = -\frac{j\omega}{\eta}\hat{r}\times\hat{r}\times\vec{A}_{ff}$

Time average power density: $\vec{S}_{ff} = \frac{1}{2}\mathrm{Re}\left\{\vec{E}_{ff}\times\vec{H}^{*}_{ff}\right\}$

Equivalent current: $\vec{J} = \frac{2\,\vec{E}_o}{\eta}$

Vacuum permittivity $\varepsilon_0 = 8.854\times10^{-12}$ F/m

Vacuum permeability $\mu_0 = 4\pi\times10^{-7}$ H/m

Intrinsic impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

Useful Integrals

$$\int \sin^2(u)du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$$

$$\int \sin^n(u)du = -\frac{1}{n}\sin^{n-1}u\,\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du$$

$$\int \cos^2(u)du = \frac{1}{2}u + \frac{1}{4}\sin(2u) + C$$

$$\int \cos^n(u)du = \frac{1}{n}\cos^{n-1}u\,\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du$$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$