ECEN 462 Exam #1

Fall 2009
Oct. 1 – Oct. 6
Prof. Stephen M. Schultz – 422-1693

Name:______________________________

Staring Time and Date:________________
Ending Time:_________________________

Instructions – Please Read

1. Closed book and closed notes
2. *All work should start from an equation in the appendix NOT start from an equation that you have memorized.*
3. 3 hour time limit
4. **Graphing Calculator Allowed**
5. Ruler and compass
6. This exam consists of 6 problems + appendix. These are all work out problems so be sure to show your work.
7. There are some charts, graphs, and equations that you might find useful at the end of the test.
8. Please make any standard class assumptions *unless otherwise indicated*. Some standard assumptions are that if $\mu_r>1$ then $\varepsilon_r=1$ and vice versa, propagation is in the z-direction, mks units, perfect conductors for metallic waveguides, charge free, lossless materials, linear, time invariant, etc… Any other assumptions should be stated.

Appendix

Time Harmonic Maxwell's Equations

$\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon \vec{E}$

$\nabla \cdot \vec{B} = 0$

$\nabla \cdot \vec{D} = \rho_v$

$\vec{D} = \varepsilon \vec{E}$

$\vec{B} = \mu \vec{H}$

$\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

$\mu_o = 4\pi \cdot 10^{-7}$ H/m

$\varepsilon_o = 8.854 \cdot 10^{-7}$ F/m

Boundary Conditions

Tangential E is continuous $\quad \hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$

Tangential H equals surface current $\quad \hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$

Normal D is surface charge $\quad \hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

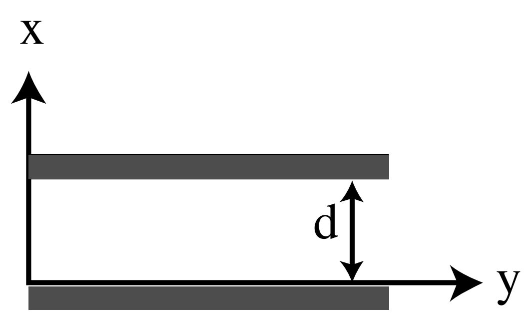
Normal B is zero $\quad \hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$

Phase Velocity: $v_p = \dfrac{\omega}{\beta}$

Group Velocity: $v_g = \left(\dfrac{\partial \beta}{\partial \omega}\right)^{-1}$

Parallel Plate Metallic Waveguides

$k_x = \dfrac{m\pi}{d}$ $\qquad k_x^2 + \beta^2 = k^2 = \left(\dfrac{\omega}{v}\right)^2 = \left(n\dfrac{\omega}{c}\right)^2$



TE Mode Fields

$\vec{E} = \hat{y}\, 2\, j\, E_o \sin(k_x x)\, e^{-j\beta z}$

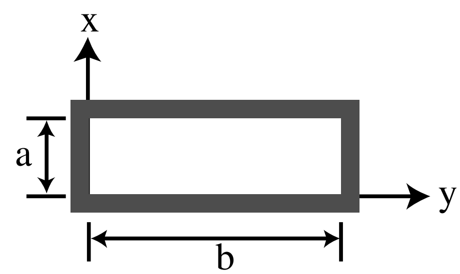
$\vec{H} = \dfrac{2 E_o}{\eta k}\left[-j\,\hat{x}\,\beta \sin(k_x x) - \hat{z}\, k_x \cos(k_x x)\right] e^{-j\beta z}$

TM Mode Fields

$\vec{E} = 2\eta H_o \left[\hat{x}\dfrac{\beta}{k}\cos(k_x x) + \hat{z}\dfrac{j k_x}{k}\sin(k_x x)\right] e^{-j\beta z}$

$\vec{H} = \hat{y}\, 2 H_o \cos(k_x x)\, e^{-j\beta z}$

Rectangular Metallic Waveguides



$$k_x = \frac{m\pi}{a}, \; k_y = \frac{n\pi}{b}$$

$$\beta = \sqrt{k^2 - k_x^2 - k_y^2} = \sqrt{\left(\frac{\omega}{v}\right)^2 - k_x^2 - k_y^2}$$

TE Mode Longitudinal Fields

$$\vec{E} = E_o \left[\hat{x}\frac{k_y}{k}\cos(k_x x)\sin(k_y y) - \hat{y}\frac{k_x}{k}\sin(k_x x)\cos(k_y y) \right] e^{-j\beta z},$$

$$\vec{H} = \frac{E_o}{\eta}\left[\hat{x}\frac{k_x \beta}{k^2}\sin(k_x x)\cos(k_y y) + \hat{y}\frac{k_y \beta}{k^2}\cos(k_x x)\sin(k_y y) - j\,\hat{z}\frac{(k_x^2 + k_y^2)}{k^2}\cos(k_x x)\cos(k_y y) \right] e^{-j\beta z}$$

TM Mode Longitudinal Fields

$$\vec{E} = -\eta H_o \left[\hat{x}\frac{k_x \beta}{k^2}\cos(k_x x)\sin(k_y y) + \hat{y}\frac{k_y \beta}{k^2}\sin(k_x x)\cos(k_y y) + j\,\hat{z}\frac{(k_x^2 + k_y^2)}{k^2}\sin(k_x x)\sin(k_y y) \right] e^{-j\beta z}$$

$$\vec{H} = H_o \left[\hat{x}\frac{k_y}{k}\sin(k_x x)\cos(k_y y) - \hat{y}\frac{k_x}{k}\cos(k_x x)\sin(k_y y) \right] e^{-j\beta z},$$

Relationships for transverse waveguide fields in Cartesian coordinates

$$H_x = \frac{j}{k^2 - \beta^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x} \right)$$

$$H_y = \frac{-j}{k^2 - \beta^2}\left(\omega\varepsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} \right)$$

$$E_x = \frac{-j}{k^2 - \beta^2}\left(\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} \right)$$

$$E_y = \frac{j}{k^2 - \beta^2}\left(-\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial x} \right)$$

Dielectric Slab Waveguides TE Modes

$$\vec{E} = \begin{cases} \hat{y}\, E_1 e^{-\alpha x}\, e^{-j\beta z} & x \geq d \\ \hat{y}\, E_o \begin{Bmatrix} \sin(k_x x) \\ \cos(k_x x) \end{Bmatrix} e^{-j\beta z} & |x| \leq d \\ \hat{y} \begin{Bmatrix} - \\ + \end{Bmatrix} E_1 e^{+\alpha x}\, e^{-j\beta z} & x \leq -d \end{cases}$$

Symmetric modes: $(\alpha\, d) = (k_y\, d) \tan(k_y\, d)$

Anti-symmetric modes: $(\alpha\, d) = -(k_y\, d) \cot(k_y\, d)$

Dispersion Equation: $(\alpha\, d)^2 + (k_y\, d)^2 = \omega^2\, \mu_o\, \varepsilon_o\, d^2\, (n_1^2 - n_2^2)$

Mode Excitation

$$P_m = -\frac{1}{2} \int_v \vec{E}_m \cdot \vec{J}\; dv$$

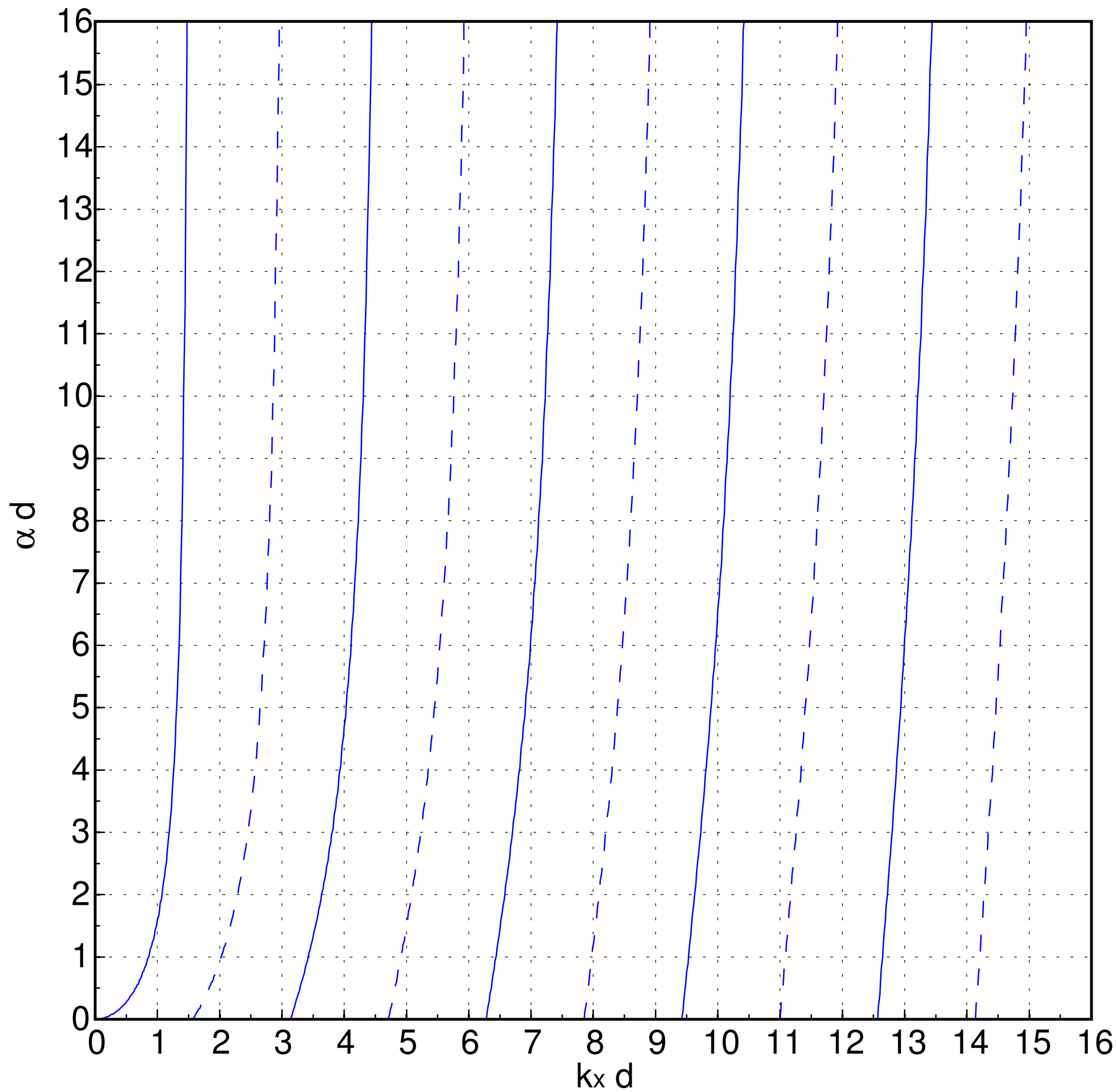


Figure 1: The plot of the guidance condition for TE modes in a dielectric waveguide. The solid lines are the equation $\alpha d = k_y d \tan(k_y d)$ and the dashed lines are the equation $\alpha d = -k_y d \cot(k_y d)$.

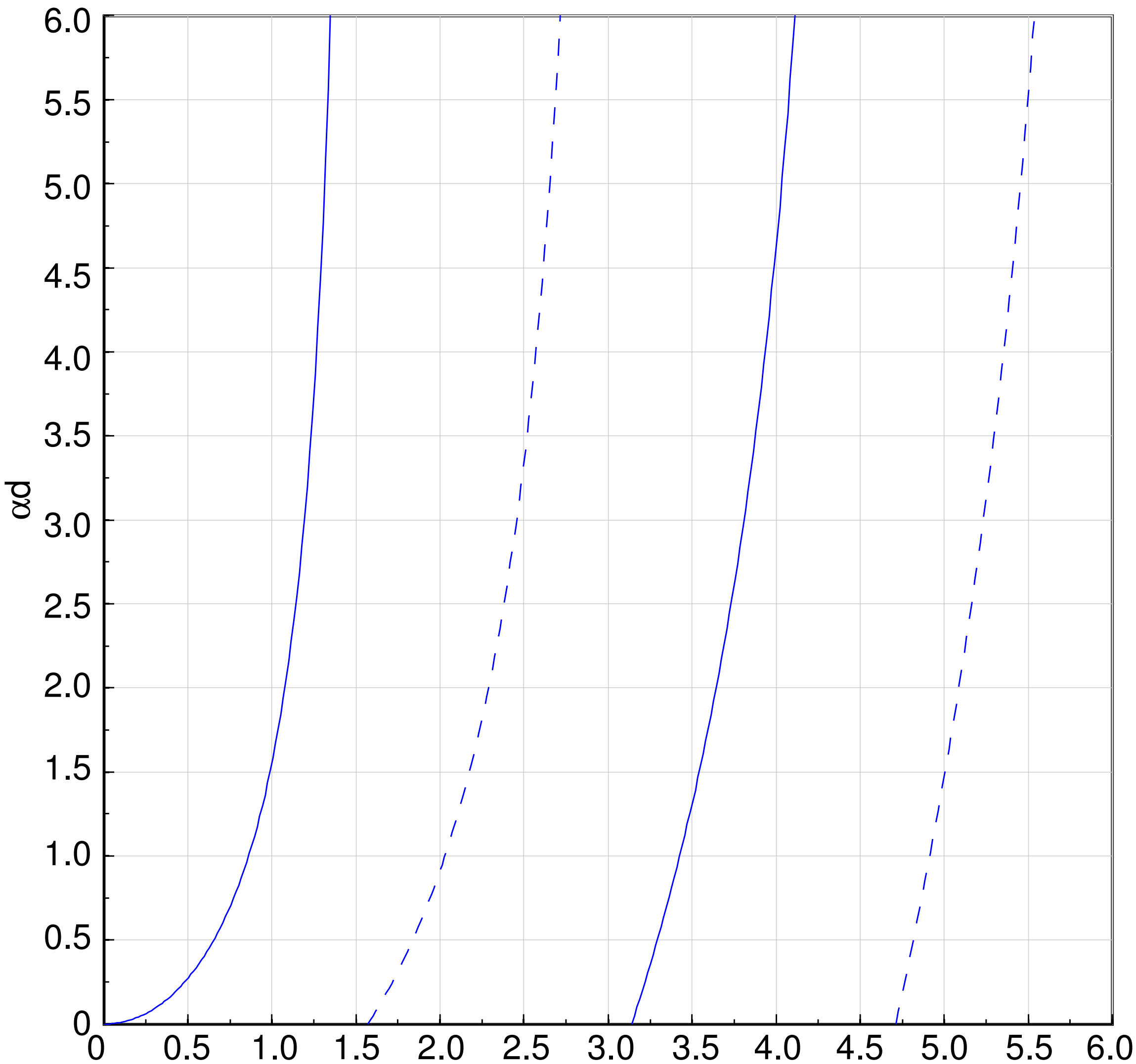


Figure 2: The plot of the guidance condition for TE modes in a dielectric waveguide. The solid lines are the equation $\alpha d = k_y d \tan(k_y d)$ and the dashed lines are the equation $\alpha d = -k_y d \cot(k_y d)$.